USN | | | | | | | | | | | **06MAT11**

First Semester B.E. Degree Examination, Dec.2013/Jan.2014

Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions only in OMR sheet page 5 of the answer booklet.***
3. ***Answer to objective type questions on sheets other than OMR will not be valued.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) $D^n\left(\frac{1}{ax+b}\right) =$

A) $\frac{(n!)a^n}{(ax+b)^{n+1}}$ B) $\frac{(-1)^n(n!)a^n}{(ax+b)^{n+1}}$ C) $\frac{(-1)^n(n!)a^{n+1}}{(ax+b)^n}$ D) $\frac{(-1)^n(n!)a^{n+1}}{(ax+b)^n}$

ii) The angle between radius vector and tangent is

A) $\tan\phi = r\frac{dr}{d\theta}$ B) $\tan\phi = \frac{1}{r}\frac{dr}{d\theta}$ C) $\tan\phi = r\frac{d\theta}{dr}$ D) $\tan\phi = \frac{dr}{d\theta}$

iii) The n^{th} derivative of $\sin(ax+b)$ is

A) $\sin\left(ax+b+\frac{n\pi}{2}\right)$ B) $a^n\cos\left(ax+b+\frac{n\pi}{2}\right)$

C) $a^n\sin\left(ax+b-\frac{n\pi}{2}\right)$ D) $a^n\sin\left(ax+b+\frac{n\pi}{2}\right)$

iv) Find ϕ, for the curve $r^2 = a^2\sin 2\theta$

A) $\phi = 2\theta$ B) $\phi = -2\theta$ C) $\phi = 4\theta$ D) $\phi = -4\theta$

b. Find the n^{th} derivative of $\cos x \cos 2x \cos 3x$. **(04 Marks)**

c. If $y = (\sin^{-1}x)^2$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$. **(06 Marks)**

d. Find the pedal equation of the parabola, $\frac{2a}{r} = 1-\cos\theta$. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) If $z = x^3 + y^3 - 3axy$, then $\frac{\partial z}{\partial x}$ is equal to

A) $+3x^2 - 3ay$ B) $-3x^2 - 3ay$ C) $3y^2 - 3ax$ D) $-3y^2 - 3ax$

ii) If $z = f(x+ct) + \phi(x-ct)$, then the value of $\frac{\partial^2 z}{\partial x^2}$ is

A) $f'(x+ct) + \phi'(x-ct)$ B) $f''(x+ct) + \phi''(x-ct)$

B) $f''(x+ct).c + \phi''(x-ct)c$ D) $f''(x+t) + \phi''(x-t)$

iii) If $f(x, y) = x^3 + y^3 + 3axy - 1$ then dy/dx is equal to

A) $\frac{x^2+y}{y^2+x}$ B) $-\frac{x^2+y}{y^2+x}$ C) $\frac{x^2-y}{y^2-x}$ D) $\frac{x^2-y}{y^2+x}$

iv) In polar co-ordinates, $x = r\cos\theta$, $y = r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)}$ is equal to

A) r^3 B) r^2 C) r D) $-r$

b. Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u\log u$ where $\log u=\frac{(x^3+y^3)}{(3x+4y)}$. **(04 Marks)**

c. If $u-F(x-y, y-z, z-x)$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. **(06 Marks)**

d. If $x=e^V\sec(u)$, $y=e^V.\tan(u)$, prove that $J\times J'=1$. **(06 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\int_0^{\pi/2}\cos^6 x\,dx$ is

A) $\frac{\pi}{32}$ B) $\frac{5\pi}{13}$ C) $\frac{5\pi}{32}$ D) $\frac{5\pi}{130}$

ii) The equation of the asymptote of $y^2(a-x)=x^2(a+x)$ is

A) $y=a$ B) $y=-a$ C) $x=-a$ D) $x=a$

iii) The curve $x=a(\theta+\sin\theta)$, $y=a(1+\cos\theta)$ is symmetrical about the

A) x - axis B) y - axis C) xy - axis D) None

iv) The value of $\int\tan^n x\,dx$ is

A) $\frac{\tan^{n-1}x}{n-1}-I_{n-2}$ B) $\frac{\tan^{n-1}x}{n-1}-I_{n-1}$ C) $\frac{\tan^{n-1}x}{n-1}+I_{n-2}$ D) $\frac{\tan^{n-1}x}{n-2}+I_{n-2}$

b. Obtain the reduction formula for $\int\sin^n x\,dx$. **(04 Marks)**

c. Evaluate $\int_0^{\pi}\theta\sin^6\theta\cos^4\theta\,d\theta$. **(06 Marks)**

d. Trace the curve $r^2=a^2\cos 2\theta$. **(06 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) The complete area of the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

A) πab^2 sq.Units B) $\pi a^2 b$ sq.Units C) πab sq.Units D) None of these

ii) Length of the polar curve $r=f(\theta)$ is

A) Length = $\int_{\theta=\alpha}^{\beta}\sqrt{\left(r^2+\left(\frac{dr}{d\theta}\right)^2\right)}d\theta$ B) Length = $\int_{\theta=\alpha}^{\beta}\left(r^2+\left(\frac{dr}{d\theta}\right)^2\right)d\theta$

C) Length = $\int_{\theta=\alpha}^{\beta}\sqrt{\left(r^2-\left(\frac{dr}{d\theta}\right)^2\right)}d\theta$ D) Length = $\int_{\theta=\alpha}^{\beta}\sqrt{\left(r+\frac{dr}{d\theta}\right)}d\theta$

iii) The volume of the solid generated by the revolution of the cardioid $r=a(1+\cos\theta)$ about the initial line is

A) $\frac{4\pi a^3}{3}$ B) $\frac{6\pi a^3}{3}$ C) $\frac{8\pi a^3}{3}$ D) $\frac{10\pi a^3}{3}$

iv) The surface area of the sphere of radius 'a' is

A) $4\pi a^2$ sq.Units B) $4\pi r$ sq.Units C) $4\pi a$ sq.Units D) $4\pi^2 a^2$ sq.Units

b. Find the entire length of the astroid, $x^{2/3}+y^{2/3}=a^{2/3}$ **(04Marks)**

c. Find the volume generated by an arc of the parabola $y^2=4ax$, from the vertex to the latus rectum about x-axis. **(06 Marks)**

d. Evaluate $\int_0^1\frac{x^\alpha-1}{\log x}dx$, $\alpha\ge 0$, using differentiation under integral sign. **(06 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The order of the differential equation, $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = C^2\left(\frac{d^2y}{dx^2}\right)^2$ is

A) 1 B) 2 C) 3 D) 4

ii) The degree of the differential equation , $y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2$ is

A) 1 B) 2 C) – 2 D) – 1

iii) The differential equation of simple harmonic motion, $\frac{d^2x}{dt^2} + n^2x = 0$ is formed from

A) $x = A\cos(nt + \alpha)$ B) $x = A\sin(nt + \alpha)$
C) $x = A\sin(nt - \alpha)$ D) $x = A\cos(nt - \alpha)$

iv) The orthogonal trajectory of the cardioids $r = a(1 - \cos\theta)$ is

A) $r = a(1 - \cos\theta)$ B) $r = a(1 - \sin\theta)$ C) $r = a(1 + \sin\theta)$ D) $r = a'(1 + \cos\theta)$

b. Solve : $\frac{dy}{dx} = (4x + y + 1)^2$ **(04 Marks)**

c. Solve : $(1 + e^{x/y})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$ **(06 Marks)**

d. Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is the parameter. **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) If $r = 1$, then the series $1 + r + r^2 + r^3 + \ldots\ldots\infty$ is

A) oscillates B) converges C) diverges D) None of these

ii) The n^{th} term of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots\ldots\infty$ is

A) $\frac{n}{n(n+1)(n+2)}$ B) $\frac{n+2}{n(n+1)(n+2)}$ C) $\frac{2n-1}{n(n+1)(n+2)}$ D) $\frac{2n+1}{n(n+1)(n+2)}$

iii) An alternating series $u_1 - u_2 + u_3 - u_4 + \cdots\cdots$ converges if

A) $u_n > u_{n+1}$ and $\lim_{n\to\infty} u_n = 0$ B) $u_n > u_{n-1}$ and $\lim_{n\to\infty} u_n \neq 0$
C) $u_n < u_{n+1}$ and $\lim_{n\to\infty} u_n = 0$ D) $u_n > u_{n+1}$ and $\lim_{n\to\infty} u_n \neq 0$

iv) The series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \ldots\ldots$ is

A) oscillatory B) absolutely convergent
C) divergent D) conditionally convergent

b. Discuss the nature of the series;

$\frac{x}{1} + \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1.3}{2.4}\cdot\frac{x^5}{5} + \frac{1.3.5}{2.4.6}\cdot\frac{x^7}{7} + \ldots\ldots\infty,\ x > 0$ **(04 Marks)**

c. Show that the series $\sum\frac{1}{n^P}$, converges if $P > 1$ and diverges if $P \leq 1$. **(06 Marks)**

d. Examine the character of the series, $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n}{2n-1}$ **(06 Marks)**

7 a. Choose the correct answers for the following : **(04 Marks)**

i) The projection of the join of two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ on the line PQ whose direction cosines are l, m, n is

A) $(x_2 - x_1)(y_2 - y_1)(z_2 - z_1)$ B) $(x_2 - x_1)+(y_2 - y_1)+(z_2 - z_1)$

C) $l(x_2 - x_1)+ m(y_2 - y_1)+ n(z_2 - z_1)$ D) $l(x_2 + x_1)+ m(y_2 + y_1)+ n(z_2 + z_1)$

ii) The angle between two diagonals of a cube is

A) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ B) $\theta = \cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ C) $\theta = \cos^{-1}\left(\frac{1}{3}\right)$ D) $\theta = \cos^{-1}(3)$

iii) The angle between the planes $2x - 3y + z + 5 = 0$ and $x + 2y + 7z - 3 = 0$ is

A) $\theta = \cos^{-1}\left(\frac{1}{2\sqrt{21}}\right)$ B) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{21}}\right)$ C) $\theta = \cos^{-1}\left(\frac{2}{\sqrt{21}}\right)$ D) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{54}}\right)$

iv) Two points form of equation of line is

A) $\frac{x + x_1}{x_2 - x_1} = \frac{y + y_1}{y_2 - y_1} = \frac{z + z_1}{z_2 - z_1}$ B) $\frac{x - x_1}{x_2 + x_1} = \frac{y - y_1}{y_2 + y_1} = \frac{z - z_1}{z_2 + z_1}$

C) $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$ D) None of these.

b. Find the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to 6, 4, – 4 and –6, 2, 1. **(04 Marks)**

c. Find the equation of the plane through (2, –1, 6) (1, –2, 4) and perpendicular to the plane $x - 2y - 2z + 9 = 0$. **(06 Marks)**

d. Find the equation of a straight line perpendicular to both the lines $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z+2}{3}$ and $\frac{x+2}{2} = \frac{y-5}{-1} = \frac{z+3}{2}$ and passing through the point of intersection. **(06 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) Find $\frac{d\vec{r}}{dt}$, for $\vec{r} = a(\cos t\,\vec{i} + \sin t\,\vec{j}) + ct\,\vec{k}$, where a and c are scalar constant.

A) $-a\cos t\,\vec{i} + \sin t\,\vec{j} + ct\,\vec{k}$ B) $-a\sin t\,\vec{i} + a\cos t j + c\,\vec{k}$

C) $-a\sin t\,\vec{i} + a\cos t j - c\,\vec{k}$ D) $-a\sin t\,\vec{i} + a\sin t\,\vec{j} + ct\,\vec{k}$

ii) If $\phi = x^3y^3z^3$, then $\nabla\phi$ at (1, 2, 1) is

A) $24\vec{i} - 12\vec{j} + 24\vec{k}$ B) $24\vec{i} + 24\vec{j} + 24\vec{k}$

C) $24\vec{i} - 2\vec{j} + 24\vec{k}$ D) $24\vec{i} + 12\vec{j} + 24\vec{k}$

iii) If $\vec{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$ then the value of div $\vec{F}$ at (1, 2, 3) is

A) 16 B) – 16 C) 32 D) – 32

iv) The value of $\nabla^2(r^n)$ is

A) $n(n-1)r$ B) $n(n+1)r^2$ C) $n(n+1)r^n$ D) $n(n+1)r^{n-2}$

b. Find the unit tangent vector to the curve $\vec{r} = \cos t\,\vec{i} + \sin t\,\vec{j} + t\,\vec{k}$. **(04 Marks)**

c. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at the point (1, –2, –1) in the direction of the vector $2i - j - 2k$. **(06 Marks)**

d. If $\vec{F} = (ax + 3y + 4z)\vec{i} + (x - 2y + 3z)\vec{j} + (3x + 2y - z)\vec{k}$ is solenoidal, find 'a'. **(06 Marks)**

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